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A MATHEMATICAL MODEL OF EXTRACTION WITH A VARIABLE MASS-TRANSFER COEFFICIENT

V. V. Beloborodov and B. A. Voronenko

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Using the relationship between the sizes of particles and the characteristics of their layer, we obtain the dependence of the mass-transfer coefficient on time and concentration; the dependence is included in the third-kind boundary condition. Using this condition, we solve the boundary-value problem of extraction.

Usually, problems of extraction are solved with a constant mass-transfer coefficient [1-10], whereas, in reality, when an extracting liquid flows through a material layer, the hydrodynamics varies over the height of the layer, and the mass-transfer coefficient is a variable quantity. The dependence of the mass-transfer coefficient on the most important factors (time, concentration) can be determined via the relationship between the size of particles and the geometric and hydrodynamic characteristics of the layer, specifically for a cylindrically-shaped one occurring in practice in connection with the wide use of such apparatuses as, for example, vertical-screw conveyers.

The extracting liquid flows through a layer of particles from the bottom upwards; then, from rather general considerations we may assume that the mass-transfer coefficient changes as an exponential function of the height of the layer and the velocity of the extracting liquid changes as a parabolic function of the radius of the layer (apparatus).

For a fixed point along the radius of the apparatus we can write

$$\beta (z)_{R_x=\mathrm{const}} = \beta_{z=0,R_x=\mathrm{const}} \exp ((2n_zR + r)/L_z) . \tag{1}$$

In deriving Eq. (1) it is assumed that the spherical particles lie on one another, while their centers are located along one vertical axis. Other models of packing [11] can be described by introducing a corresponding coefficient. As z grows, the coefficient β increases, since in this case the looseness (porosity) increases (there is a greater tendency toward caking of particles at the bottom and they are less washed away by the extracting liquid flow). Therefore the exponential in Eq. (1) has a plus sign.

It is evident that

$$2n_zR = W(R_x)\,\tau_x . \tag{2}$$

Since $\tau = \tau_x n$, then

$$2n_zR = W(R_x)\,\frac{\tau}{n}, \tag{3}$$

$$n = \frac{v_{\mathrm{liq}}}{m v_{\mathrm{mat}}} . \tag{4}$$

According to the statement of the problem, the liquid velocity is distributed parabolically and therefore

$$W(R_x) = W(0) + B(W_n)\left(\frac{R_x}{R_a}\right)^2 . \tag{5}$$

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Since the centers of the adjacent particles are located along one horizontal axis, then $R_x = 2k_xR$.

With allowance for Eqs. (2)-(5) we obtain from Eq. (1)

$$\beta(\tau)_{R_x=\mathrm{const}} = e\beta_{z=0,R_x=\mathrm{const}} \exp(E\tau/F) \tag{6}$$

or

$$\beta(\tau)_{R_x=\mathrm{const}} = a_1 \exp(b\tau). \tag{7}$$

In Eq. (1) $2n_zR + r \leq L_z$ and therefore the value of $\beta(z)_{R_x=\mathrm{const}}$ does not increase infinitely; it increases by a factor of e on one passage of the extracting liquid through the layer. When passing to the dependence of the mass-transfer coefficient on time, i.e., to Eq. (7), we must take the first emergence of the extracting liquid from the layer as the beginning of the process (at this time all the particles of the layer come in contact with the extracting liquid), i.e., now $\beta(0)_{R_x=\mathrm{const}} = a_1$ (e times higher than at the entry into the layer). In the case of multiple (continuous) passage of liquid through the layer, Eq. (7) is inconsistent with physical concepts, since the mass-transfer coefficient cannot increase infinitely. It is evident that under these conditions we may write the equation

$$\beta(\tau)_{R_x=\mathrm{const}} = a_1 + a_2 - a_2 \exp(-b\tau) = a - a_2 \exp(-b\tau), \tag{8}$$

which corresponds to the physical picture: $\beta(\tau)_{R_x=\mathrm{const}} \to a_1$, when $\tau \to 0$ and $\beta(\tau)_{R_x=\mathrm{const}} \to a_1 + a_2$, when $\tau \to \infty$; $a_1 > 0$, $a_2 > 0$, where a_2 is the mass-transfer coefficient component determined by the change in the concentration of the extracting liquid in the course of the process; $b = \mathrm{const} > 0$.

Thus, the formulation of the problem for spherical particles can be written as follows:

$$\frac{\partial C(r,\tau)}{\partial \tau} = D\left(\frac{\partial^2 C(r,\tau)}{\partial r^2} + \frac{2}{r}\frac{\partial C(r,\tau)}{\partial r}\right), \quad 0 < r < R, \quad \tau > 0; \tag{9}$$

$$C(r, 0) = C_0 = \mathrm{const}; \tag{10}$$

$$\frac{\partial C(0,\tau)}{\partial r} = 0; \quad C(0,\tau) < \infty; \quad -\frac{\partial C(R,\tau)}{\partial r} - \beta(\tau)_{R_x=\mathrm{const}} C(R,\tau) = 0. \tag{11}$$

The solution of the posed problem according to the procedure of [12] is

$$\Theta(X, \mathrm{Fo}) = \frac{C(r,\tau) - C_0}{C_0 \mathrm{Po}} = 3\sum_{n=0}^{\infty} A_n \left[\frac{k_1}{n}(1 - \exp(-n\,\mathrm{Po}\,\mathrm{Fo})) - \right.$$

$$\left. - \frac{k_2}{n}(1 - \exp(-(1+n))\,\mathrm{Po}\,\mathrm{Fo}\right] + \frac{3}{X}\sum_{n=0}^{\infty}\sum_{l=1}^{n} B_{n,l}\sin(\sqrt{l}\,X)\exp(-l\,\mathrm{PoFo}) \times$$

$$\times \left[\frac{k_1}{nl}(1 - \exp(-nl\,\mathrm{PoFo})) - \frac{k_2}{1+nl}(1 - \exp(-(1+nl))\,\mathrm{PoFo})\right] +$$

$$+ \frac{2}{X}\sum_{n=0}^{\infty}\sum_{l=0}^{n}\sum_{m=1}^{\infty} C_{n,l,m}\sin(\sqrt{\mu_m^2 + l^2}\,X) \times$$

$$\times \exp(-(\mu_m^2 + l)\,\mathrm{Fo})\left[\frac{k_1}{n - \mu_m^2 - l}(1 - \exp(-(n - \mu_m^2 - l))\,\mathrm{Fo}) - \right.$$

$$- \frac{k_2}{1 + (n - \mu_m^2 - l)} (1 - \exp(-(1 + (n - \mu_m^2 - l)) \mathrm{PoFo}))\Bigg] . \tag{12}$$

Here $X = r/R$; $\mathrm{Fo} = D\tau/R^2$; $\mathrm{P_0} = bR^2/D$; $k_1 = aR$; $k_2 = a_2R$; μ_m are successive positive roots of the characteristic equation

$$\tan\mu = \frac{\mu}{1 - k_1}; \tag{13}$$

$$A_n = \frac{(-k_2)^n}{\prod\limits_{k=1}^{n} (1 - k_1 - \sqrt{k} \operatorname{cth} \sqrt{k})};$$

$$B_{n,l} = \frac{(-k_2)^n \prod\limits_{\substack{k=1 \\ k \neq l}}^{n} \sin\sqrt{l-k}}{\prod\limits_{\substack{k=0 \\ k \neq l}}^{n} [-\sqrt{l-k}\cos\sqrt{l-k} + (1 - k_1)\sin\sqrt{l-k}\,]};$$

$$C_{n,l,m} = \left[(-k_2)^n \prod_{k=1}^{n} \sin\sqrt{\mu_m^2 + l - k}\right] \Big/ D_{n,l,m};$$

$$D_{n,l,m} = \sin\mu_m \prod_{\substack{k=0 \\ k \neq l}}^{n} [-\sqrt{\mu_m^2 + l - k}\cos\sqrt{\mu_m^2 + l - k} + (1 - k_1)\sin\sqrt{\mu_m^2 + l - k}\,].$$

Solution (12) is obtained for a mass-transfer coefficient that changes exponentially and for an extracting liquid velocity that changes parabolically. The basic idea of the present work is the relationship between the size of particles and the characteristics of the layer. Therefore, solutions of the extraction problem with other laws for the dependence of the mass-transfer coefficient and velocity would have been only modifications of the basic idea and would not have introduced fundamental innovations.

NOTATION

C, concentration; r, current radius of a particle; R, radius of a particle; L_z and L_x, current height and radius of the layer (apparatus); L_z and R_a, height of the layer and radius of the apparatus; n_z, number of particles up to the height z at $R_x = \mathrm{const}$; τ, time; D, diffusion coefficien; β, mass-transfer coefficient; $W(R_x)$, liquid velocity at the points $R_x = \mathrm{const}$; $W(0)$, liquid velocity at $R_x = 0$; τ_x, time of the liquid motion to the point R_x; n, number of passages of the liquid through the layer for the time τ; v_{liq}, volumetric flow rate of the liquid; v_{mat}, volumetric flow rate of the material; m, porosity of the material layer; k_x, number of particles over the radius of the layer (apparatus) up to the given point $R_{x,z}$; $B(W_n)$, coefficient; a, a_1, a_2, b, coefficients (see the text); l, number of the term of the corresponding sum; $E = [W(0) + 4B(W_0)(k_x^2R^2/R_x)]mG$; $F = v_{\mathrm{liq}}\rho L_z$; $a_1 = e\beta_{z=0,R_x=\mathrm{const}}$; $b = E/F$.

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